ESE 2026
Main Examination

UPSC ENGINEERING SERVICES EXAMINATION

Topicwise

Conventional Practice Book

Mechanical Engineering

PAPER-I

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1 Thermodynamics

1. Basic Concepts & Zeroth law of thermodynamics

Level-1

1.1 The temperature t on a thermometric scale is defined in terms of a property k by the relation

$$t = a \ln k + b$$

where a and b are constants. The values of k are found to be 1.83 and 6.78 at the ice point and the steam point, the temperature of which are assigned the numbers 0 to 100 respectively. Determine the temperature corresponding to a reading of k equal to 2.42 on the thermometer.

(10 Marks)

Solution:

Given : $k_0 = 1.83$ at $t = 0°$, $k_{100} = 6.78$ at $t = 100°$

To find, $t = ?$ at $k = 2.42$

$$0 = a \ln 1.83 + b$$

$$100 = a \ln 6.78 + b \quad \text{...(ii)}$$

(ii) – (i) $100 = a(\ln 6.78 - \ln 1.83)$

$\therefore$ $a = \dfrac{100}{1.3097}$

$\therefore$ $a = 76.356$

So, from (i) $b = -a \ln 1.83 = -46.143$

at $k = 2.42$,

$\therefore$ $t = a \ln k + b = 76.356 \ln(2.42) - 46.143$

$t = 21.338°$

1.2 The resistance of the windings in a certain motor is found to be 80 ohms at room temperature (25°C), when operating at full load under steady state conditions. The motor is switched off and the resistance of the windings immediately measured again and is found to be 93 ohms. The windings are made of copper whose resistance at temperature t°C is given by

$$R_t = R_0(1 + 0.00393\ t)$$

Where R_0 is the resistance at 0°C. Find the temperature attained by the coil during full load.

(10 Marks)

Solution:

Given, $R_t = 80\ \Omega$ at $t = 25°C$, $R_t = 93\ \Omega$ at $t = t_1$, $R_t = R_0(1 + 0.00393t)$, $80 = R_0(1 + 0.00393 \times 25)$

$\therefore$ $R_0 = \dfrac{80}{1.09825}$

$\therefore$ $R_0 = 72.843\ \Omega$

$\therefore$ $R_t = R_0(1 + 0.00393\ t)$

$93 = 72.843(1 + 0.00393\ t_1)$

$1 + 0.00393\ t_1 = 1.2767$

$\therefore$ $t_1 = 70.411°C$

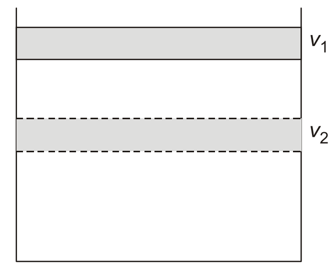
2. Energy & its Interaction

Level-1

2.1 A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa. for which pv = constant. The initial density of air is 1.16 kg/m^3. Find the work done by the piston to compress the air.

(10 Marks)

Solution:



Given :

$$m = 1.5 \text{ kg}, \quad \rho_1 = 1.16 \text{ kg/m}^3,$$

$$v_1 = \frac{V_1}{m} = \frac{1}{\rho_1}$$

$$P_1 = 0.1 \text{ MPa}, \quad P_2 = 0.7 \text{ MPa}$$

For air, as an ideal gas,

$$Pv = RT \quad \text{...(i)}$$

and

$$w = \int_1^2 Pdv$$

from (i),

$$p_1 v_1 = P_2 v_2 = \text{constant} \quad \text{... (ii)}$$

$$0.1 \times 10^6 \times \frac{1}{1.16} = \text{const} = k(\text{Let}) \qquad \left[P_1 \times \frac{1}{\rho_1} = k\right]$$

$$\therefore \quad k = 0.086 \times 10^6$$

$$\therefore \quad w = \int_1^2 \frac{k}{v} dv = k \ln v \Big]_1^2 = k \ln \frac{v_2}{v_1} = k \ln\left(\frac{P_1}{P_2}\right) \qquad \text{From (ii)}$$

$$w = 0.086 \times 10^6 \ln\left(\frac{0.1}{0.7}\right) = -0.16775 \times 10^6 \text{ J/kg.}$$

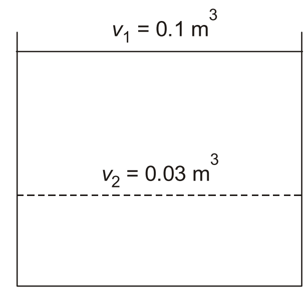
$$\therefore \quad W = m \times w = 1.5 \times -0.16775 \times 10^3 \text{ kJ} = = -251.626 \text{ kJ}$$

$$\therefore \quad \text{work done by piston} = 251.626 \text{ kJ}$$

2.2 A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. Assuming that the pressure and volume are related by PV^n = constant? Find the work done by the gas system.

(10 Marks)

Solution:



Given :

$$P_1 = 80 \text{ kPa}$$

$$P_2 = 0.4 \text{ MPa} = 400 \text{ kPa}$$

$$V_1 = 0.1 \text{ m}^3$$

$$V_2 = 0.03 \text{ m}^3$$

$$Pv^n = \text{constant}$$

$$P_1 V_1^n = P_2 V_2^n$$

$$80 \times 0.1^n = 400 \times 0.03^n$$

$$\left(\frac{10}{3}\right)^n = 5$$

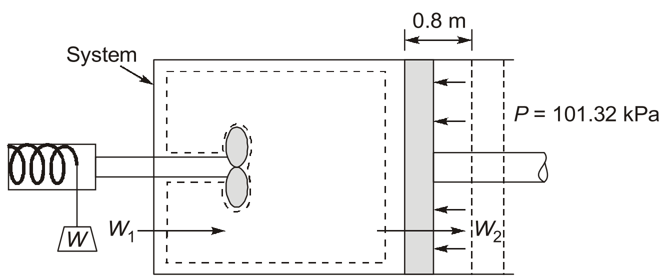
$\therefore \quad n = 1.3368$

$$\therefore \quad W = \int P dv = \int \frac{C}{v^n} dv = \left.\frac{cv^{1-n}}{1-n}\right]_{v_1}^{v_2} = \frac{P_1 v_1 - P_2 v_2}{n-1}$$

$$= \frac{80 \times 0.1 - 400 \times 0.03}{1.3368 - 1}$$

$$W = -11.876 \text{ kJ}$$

2.3 A piston and cylinder machine containing a fluid system has a string device in the cylinder. The piston is frictionless, and it is held down against the fluid due to the atmospheric pressure of 101.325 kPa. The string device is turned 10,000 revolutions with an average torque against the fluid of 1.275 Nm. Meanwhile the piston of 0.6 m diameter moves out 0.8 m. Find the net work transfer for the system.



(10 Marks)

Solution:

Work done by the string device upon the system.

$$W_1 = 2\pi TN = 2\pi \times 1.275 \times 10000 \text{ Nm}$$
$$= 80 \text{ kJ}$$

This is negative work for the system, work done by the system upon the surrounding

$$W_2 = (pA).L = 101.325 \times \frac{\pi}{4}(0.6)^2 \times 0.80 = 22.9 \text{ kJ}$$

This is positive work for the system. Hence, the work transfer for the system

$$W = W_1 + W_2$$
$$= -80 + 22.9$$
$$W = -57.1 \text{ kJ.}$$

2.4 To a closed system 150 kJ of work is supplied. If the initial volume is 0.6 m^3 and pressure of the system changes as $P = 8 - 4V$, where P is in bar and V is in m^3, determine the final volume and pressure of the system.

(10 Marks)

Solution:

Amount of work supplied to closed system = 150 kJ

initial volume = 0.6 m^3

pressure volume relation ship, $P = 8 - 4V$

The work done during the process is given by,

$$W = \int_{V_1}^{V_2} P dV = 10^5 \int_{0.6}^{V_2} (8 - 4V)\, dV = 10^5 \left[8V - 4 \times \frac{V^2}{2}\right]_{0.6}^{V_2}$$

$$= 10^5 \left[8(V_2 - 0.6) - 2(V_2^2 - 0.6^2)\right] = 10^5 (8V_2 - 2V_2^2 - 4.08) \text{ Nm}$$

as this work is supplied to the system.

$$\therefore \quad -150 \times 10^3 = 10^5(8V_2 - 2V_2^2 - 4.08)$$

$$2V_2^2 - 8V_2 + 2.58 = 0$$

$$V_2 = \frac{8 \pm \sqrt{64 - 4\times 2\times 2.58}}{4} = \frac{8 \pm 6.585}{4}$$

$$V_2 = 0.354 \text{ m}^3$$

Positive sign is incompatible with the present problem, therefore it is not considered.

$$\text{Final volume, } V_2 = 0.354 \text{ m}^3$$

$$\text{Final pressure, } P_2 = 8 - 4\text{ V} = 8 - 4 \times 0.354$$

$$P_2 = 6.584 \text{ bar}$$

$$P_2 = 6.584 \times 10^5 \text{ N/m}^2 \text{ or Pa}$$

2.5 A cylinder contains 1 kg of certain fluid of an initial pressure of 20 bar. The fluid is allowed to expand reversibly behind a piston according to a law PV^2 = constant until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original volume, heat is then supplied reversibly with the piston firmly locked in position until the pressure rises to the original value of 20 bar. Calculate the net work done by the fluid, for an initial volume of 0.05 m^3.

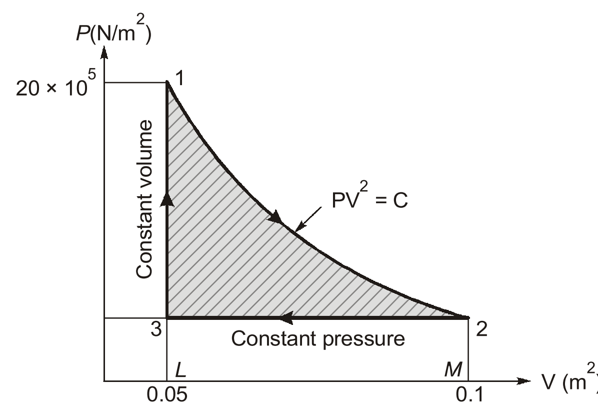
(10 Marks)

Solution:

Mass of fluid, m= 1 kg

$$P_1 = 20 \text{ bar} = 20 \times 10^5 \text{ N/m}^2$$

$$V_1 = 0.05 \text{ m}^3$$



Considering the process 1 – 2

$$P_1V_1^2 = P_2V_2^2$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^2 = 20\left(\frac{V_1}{2V_1}\right)^2 (\because V_2 = 2V_1 \text{ (given)})$$

$$= \frac{20}{4} = 5 \text{ bar}$$

work done by the fluid from 1 to 2 = Area under = $\int_1^2 \text{Pdv}$

$$W_{1\text{-}2} = \int_{V_1}^{V_2} \frac{C}{V^2}\text{dv}$$

Where, $C = P_1V_1^2 = (20 \times 10^5)(0.05)^2 \text{ Pa-m}^6$

$$\therefore \quad W_{1\text{-}2} = 10^5 \times 20 \times 0.0025\left[-\frac{1}{V}\right]_{0.05}^{0.1}$$

$$= 10^5 \times 20 \times 0.0025\left(\frac{1}{0.05} - \frac{1}{0.1}\right) = 50000 \text{ Nm}$$

work done on fluid from 2 to 3

$$= \text{Area under 2-3} = P_2(V_2 - V_3)$$

$$= 10^5 \times 5 \times (0.1 - 0.05) = 25000 \text{ Nm}$$

Work done during the process 3 – 1

= 0, because piston as locked in pistons
(Volume remains constant)

$\therefore$ Net work done by the fluid = Enclosed area 1231 = 50000 – 25000

= 25000 Nm

Level-2

2.6 A fluid undergoes the following processes.

(i) Heated reversibly at a constant pressure of 1.05 bar until it has a specific volume of 0.1 m^3/kg.

(ii) It is then compressed reversibly according to a law Pv = constant to a pressure of 4.2 bar.

(iii) It is then allowed to expand reversibly according to a law $Pv^{1.3}$ = constant.

(iv) Finally it is heated at constant volume back to initial conditions.

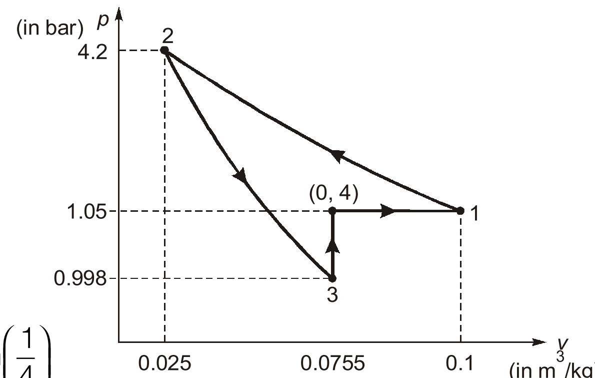
The work done in the constant pressure process is 515 Nm and the mass of fluid present is 0.2 kg. Calculate the net work done on or by the fluid in the cycle and sketch the cycle on P-v diagram.

(20 Marks)

Solution:

Given: m = 0.2 kg, $P_0 = P_1$ = 1.05 bar = 105 kPa, v_1 = 0.1 m^3/kg, w_1 = 515 Nm, P_2 = 4.2 bar, $v_3 = v_4$

State 0 = state 4



for process 0-1, constant pressure heat addition,

$$W_1 = mP_0 (v_1 - v_0)$$

$$515 = 0.2 \times 105 \times 10^3 (0.1 - v_0)$$

$$v_0 = 0.0755 \text{ m}^3/\text{kg}$$

For process 1-2, $P_1 v_1 = P_2 v_2$

$$1.05 \times 0.1 = 4.2 \times v_2$$

$$\therefore \quad v_2 = 0.025 \text{ m}^3/\text{kg}$$

$$\therefore \quad W_2 = mP_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = 0.2 \times 105 \times 0.1 \ln\left(\frac{1}{4}\right)$$

$$= -2.9112 \text{ kNm} = -2911.2 \text{ Nm.} \qquad \text{... (i)}$$

For process 3-4, constant volume heat rejection

$$\dot{W}_4 = 0 \qquad \text{... (ii)}$$

process 2-3, $Pv^{1.3}$ = constant

$$P_2 v_2^{1.3} = P_3 v_3^{1.3}$$

$$420 \times (0.025)^{1.3} = P_3 (0.0755)^{1.3}$$

$$\therefore \quad P_3 = 99.825 \text{ kPa}$$

$$\therefore \quad \dot{W}_3 = \frac{\dot{m}(P_2 v_2 - P_3 v_3)}{n-1} = \frac{0.2(420 \times 0.025 - (99.82 \times 0.0755))}{(1.3-1)}$$

$$= 1.9755 \text{ kNm} = 1975.5 \text{ Nm} \qquad \text{... (iii)}$$

$$\therefore \quad \text{Net work} = W_{net} = \dot{W}_1 + \dot{W}_2 + \dot{W}_3 + \dot{W}_4 \qquad \text{[from (i), (ii) and (iii)]}$$

$$= 515 - 2911.2 + 1975.5 + 0$$

$$\dot{W}_{net} = -420.7 \text{ Nm}$$

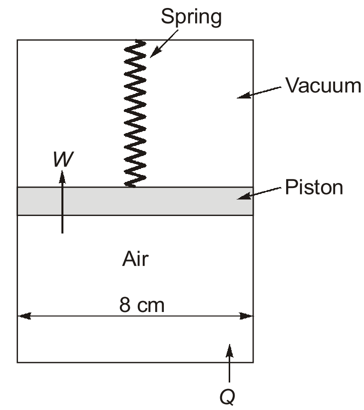
2.7 Figure shows a cylinder of 8 cm inside diameter having a piston loaded with a spring (stiffness 150 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3×10^5 N/m^2, 0.000045 m^3 and 20°C respectively. Determine the amount of heat added to the system. So that piston moves by 3.5 cm. Assume C_v = 0.717 kJ/kgK and R = 0.287 kJ/kgK.

(20 Marks)

Solution:

Given: D = 8 cm, k = 150 N/cm, P_1 = 300 kPa, V_1 = 0.000045 m^3, T_1 = 20°C, ΔL = 3.5 cm

Initial condition,



$$V_1 = 0.000045 \text{ m}^3$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\times 0.08^2$$

$$= 5.0265 \times 10^{-3} \text{ m}^2$$

$$\therefore \quad L_1 = \frac{V_1}{A} = 0.00895 \text{ m}$$

$$\therefore \quad L_2 = 0.00895 + 0.035 = 0.04395$$

$$\therefore \quad V_2 = L_2 A = 0.0002209 \text{ m}^3$$

$$P_1 = 300 \text{ kPa}$$

$$P_2 A = P_1 A + k(L_2 - L_1)$$

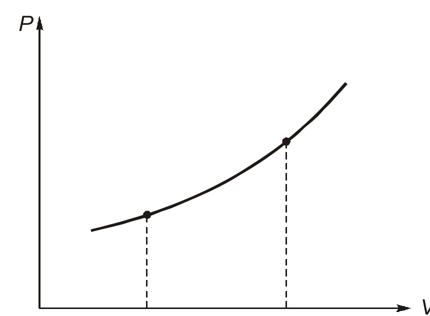
$$\therefore \quad P_2 = P_1 + \frac{k}{A}(L_2 - L_1)$$

$$= 300\times 10^3 + \frac{15000}{5.0265\times 10^{-3}}\times 0.035 = 404.446 \text{ kPa}$$

$$\therefore \quad W = \left(\frac{P_1+P_2}{2}\right)(V_2 - V_1)$$

$$= \left(\frac{300+404.446}{2}\right)(220.9-45)\times 10^{-6}$$

$$W = 61.956 \text{ J}$$



Now,
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad T_2 = \frac{404.446\times 0.0002209}{300\times 0.000045}\times 293$$

$$= 1939.3 \text{ K}$$

$$m = \frac{P_1V_1}{RT_1} = \frac{300\times 45\times 10^{-6}}{0.287\times 293} = 0.0001605 \text{ kg}$$

$$\therefore \quad \Delta U = mC_v(T_2 - T_1)$$

$$= 0.0001605 \times 0.717(1939.3 - 293)$$

$$= 0.1845 \text{ kJ} = 189.5 \text{ J}$$

$$\therefore \quad Q = \Delta U + W = 251.456 \text{ J}$$

2.8 A system consisting of 1 kg of an ideal gas at 5 bar pressure and 0.02 m^3 volume executes a cyclic processe comprising the following three distinct operations : (i) Reversible expansion to 0.08 m^3 volume and 1.5 bar pressure, pressuring pressure to be a linear function of volume ($P = a + bV$), (ii) Reversible cooling at constant pressure and (iii) Reversible hyperbolic compression according to law PV = constant. This brings the gas back to initial conditions.

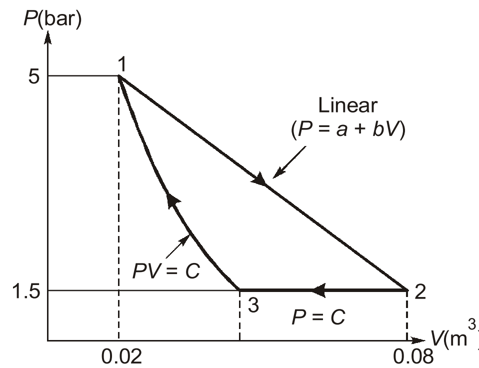
(i) Sketch the cycle on P-V diagram.

(ii) Calculate the work done in each process stating whether it is done on or by the system and evaluate the net cyclic work and heat transfer. (20 Marks)

Solution:

Given : m = 1 kg, P_1 = 5 bar, V_1 = 0.02 m^3, V_2 = 0.08, P_2 = 1.5 bar

(i) P-V diagram :



(ii) Work done and heat transfer

Process 1 – 2 (linear law) :

$$P = a + bV$$

The value of constant a and b can be determined from the value of pressure and volume at the state points 1 and 2.

$$5 = a + 0.02\, b \quad ...(i)$$

$$1.5 = a + 0.08\, b \quad ...(ii)$$

from equation (i) and (ii), we get

$$b = -58.33 \text{ and } a = 6.167$$

$$W_{1-2} = \int_1^2 PdV = \int_1^2 (a + bV)dV = \int_1^2 (6.167 - 58.33\,V)\, dV$$

$$= 10^5 \left| 6.167\,V - 58.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08}$$

$$= 10^5 \left| 6.167(0.08 - 0.02) - 58.33 \times \frac{(0.08^2 - 0.02)^2}{2} \right| \times 10^{-3} \text{kJ}$$

$$W_{1-2} = 19.5 \text{ kJ}$$

Process 2–3 (constant pressure) :

$$P_3 = P_2 = 1.5 \text{ bar}$$

The volume V_3 can be worked out from the hyperbolic compression 3–1, as follows:

$$P_1V_1 = P_3V_3$$

or

$$V_3 = \frac{P_1V_1}{P_3} = \frac{5 \times 0.02}{1.5} = 0.0667 \text{ m}^3$$

Now work for 2-3 process,

$$W_{2\text{-}3} = P_2 [V_3 - V_2] = (1.5)(10^5)[0.0667 - 0.08]$$

$$= -1995 \text{ J} = -1.995 \text{ kJ}$$

Process 3 – 1(Hyperbolic process)

$$W_{3-1} = P_3V_3 \ln\left(\frac{V_1}{V_3}\right) = (10^5 \times 1.5) \times 0.0667 \ln\left(\frac{0.02}{0.0667}\right) \times 10^{-3} \text{kJ}$$

$$W_{3-1} = -12.05 \text{ kJ}$$

$$\text{New work done, } W_{net} = W_{1-2} + W_{2-3} + W_{3-1}$$

$$= 19.5 + (-1.995) + (-12.05) = 5.455 \text{ kJ}$$

Heat transferred during the complete cycle,

$$\oint \delta Q = \oint \delta W = 5.455 \text{ kJ}$$

3. First law of thermodynamics

Level-1

3.1 A certain gas of mass 4 kg is contained within a piston cylinder assembly. The gas undergoes a process for which $PV^{1.5}$ = constant. The initial state is given by 3 bar, 0.1 m³. The change in internal energy of the gas in the process is $u_2 - u_1 = -4.6$ kJ/kg. Find the net heat transfer for the process when the final volume is 0.2 m³. Neglect the changes in K.E and P.E.

(10 Marks)

Solution:

Given: $m = 4$ kg, $P_1 = 3$ bar, $V_1 = 0.1$ m³, $V_2 = 0.2$ m³, $\Delta U = -4.6$ kJ/kg

For a process $PV^{1.5}$ = constant

$$P_1V_1^{1.5} = P_2V_2^{1.5}$$

$$300 \times (0.1)^{1.5} = P_2 \times (0.2)^{1.5}$$

$$\therefore \quad P_2 = 106.066 \text{ kPa}$$

$$\therefore \quad \Delta W = \int PdV = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{300\times0.1 - 106.066\times0.2}{1.5-1} = 17.574 \text{ kJ}$$

$$\therefore \quad \Delta U = \dot{m}\times(-4.6) = 4 \times (-4.6) = -18.4 \text{ kJ}$$

$$\therefore \quad Q = \Delta U + \Delta W = -18.4 + 17.574 = -0.826$$

$$Q = -0.826 \text{ kJ}$$

3.2 A system undergoes a process 1-2 in which it absorbs 200 kJ energy as heat while it does 100 kJ work. Then it follows the path 2-3 in which it rejects 50 kJ energy as heat when 80 kJ work is done on it. If it is required to restored the system to state 1 through an adiabatic path, calculate the work and heat interactions along the adiabatic path. Also calculate the net work and heat interaction.

(10 Marks)

Solution:

Application of the first law of thermodynamics to the process 1-2 gives.

$$U_2 - U_1 = Q_{12} - W_{12} = 200 - 100 = 100 \text{ kJ}$$

$$U_3 - U_2 = Q_{23} - W_{23} = -50 - (-80) = 30 \text{ kJ}$$

For the complete cycle $\Delta U = 0$

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

$$100 + 30 + (U_1 - U_3) = 0$$

or $$U_1 - U_3 = -130 \text{ KJ}$$

The process 3 – 1 is desired to be adiabatic therefore $Q_{31} = 0$

The first law of thermodynamics for the process 3 – 1, gives,

$$U_1 - U_3 = Q_{31} - W_{31}$$

or $$-130 = 0 - W_{31}$$

or $$W_{31} = 130 \text{ kJ}$$

Therefore, work done during the adiabatic process = 130 kJ

$$\text{Net work, } W = W_{12} + W_{23} + W_{31} = 100 + (-80) + 130 = 150 \text{ kJ}$$

We know that for a cyclic process, $\oint \delta Q = \oint \delta W = 150$ kJ

Therefore, net heat interaction = 150 kJ

Level-2

3.3 A piston-cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops and the enclosed volume is 400 L. The mass of the piston is such that a 350 kPa pressure is required to move it. The air is now heated until its volume is doubled. Determine

(a) The final temperature

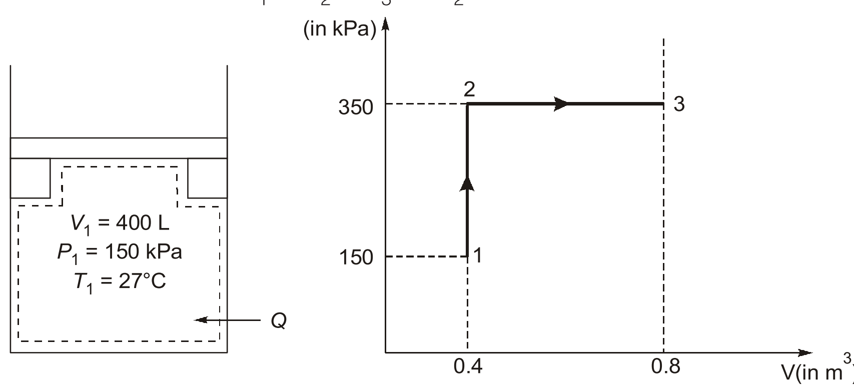
(b) The work done by air

(c) The total heat transferred to the air. [$\gamma = 1.4$] (20 Marks)

Solution:

Given :

$$P_1 = 150 \text{ kPa}$$
$$T_1 = 27°\text{C} = 300 \text{ K}$$
$$V_1 = 400 \text{ L} = 0.4 \text{ m}^3$$
$$V_1 = V_2; \quad V_3 = 2V_2$$



for process 1–2, till pressure reaches 350 kPa.

i.e. constant volume process, $W_{1-2} = 0$,

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}$$

$$\therefore \quad T_2 = \frac{350}{150} \times 300 = 700 \text{ K}$$

$$U_{1-2} = mc_v(T_2 - T_1) = (P_2 - P_1)\frac{V}{\gamma - 1} = 200 \times \frac{0.4}{0.4} = 200 \text{ kJ}$$

$$Q_{1-2} = U_{1-2} + W_{1-2} = 200 \text{ kJ}$$

for process 2-3, constant pressure,

$$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$\therefore \quad T_3 = 2 \times 700 = 1400 \text{ K}$$

(a) $\therefore$ final temperature $T_3 = 1400$ K

(b)
$$W_{2-3} = P_2(V_3 - V_2) = 350(0.8 - 0.4) = 140 \text{ kJ}$$

$\therefore$ Work, $W = W_{1-2} + W_{2-3} = 0 + 140 = 140$ kJ

(c) $$U_{2-3} = mc_v(T_3 - T_2) = \frac{P_2(V_3 - V_2)}{\gamma - 1} = 350 \times \frac{0.4}{0.4} = 350 \text{ kJ}$$

$\therefore$ Net increase in internal energy, $U_{1-3} = 200 + 350 = 550$ kJ

$\therefore$ $Q = W + U_{1-3} = 140 + 550 = 690$ kJ

3.4 In a piston-cylinder assembly some mole of air is compressed from the initial state 300 K and 1 bar till its volume is reduced to 1/15 of its original volume. The compression process can be approximated as $PV^{1.25}$ = constant. Determine:

(a) The pressure and temperature of the air at the end of the compression process.

(b) Work done on the air

(c) The energy transferred as heat. The air may be treated as an ideal gas. (15 Marks)

Solution:

(a) The polytropic process follows the relation

$$PV^{1.25} = \text{constant}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{1.25}$$

or $$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.25}$$

$$P_2 = 1 \times (15)^{1.25} = 29.52 \text{ bar}$$

For ideal gas, $$T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}$$

or, $$T_2 = \frac{29.52 \times 10^5}{1 \times 10^5}\left(\frac{1}{15}\right) \times 300 = 590.4 \text{ K}$$

(b) work done, $$w = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

$$= \frac{8.314(300 - 590.4)}{1.25 - 1} = -9657.5 \text{ J/mol}$$

(c) The first law of thermodynamics gives

$$du = dq - dw$$

or $u_2 - u_1 = q - w$ [Here, $\gamma = 1.4$, Ratio of specific heats of air]

or $c_v(T_2 - T_1) = q - w$

or $$\frac{R}{\gamma - 1}(T_2 - T_1) = q - w$$

$$q = \frac{R(T_2 - T_1)}{\gamma - 1} + w = \frac{8.314(590.4 - 300)}{1.4 - 1} + (-9657.5)$$

$$q = -3621.5 \text{ J/mol.}$$

3.5 A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume ($P = a + bV$). The internal energy of the fluid is given by the following equation:

$$U = 34 + 3.15\ PV$$

where U is in kJ, P in kPa, and V in cubic metre.

If the fluid changes from an initial state of 170 kPa, 0.03 m^3 to a final state of 400 kPa, 0.06 m^3, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer. (15 Marks)

Solution:

The change in the internal energy of the fluid during process.

$$U_2 - U_1 = 3.15\,(P_2V_2 - P_1V_1)$$
$$= 315(4 \times 0.06 - 1.7 \times 0.03) = 315 \times 0.189 = 59.5 \text{ kJ} \quad \text{... (i)}$$

Now, $P = a + bV$

$$170 = a + b \times 0.03$$
$$400 = a + b \times 0.06$$

from these two equation, $a = -60 \text{ kN/m}^2$

$$b = 7667 \text{ kN/m}^5$$

work transfer involved during the process

$$W_{1-2} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} (a + bV)dV = a(V_2 - V_1) + b\frac{V_2^2 - V_1^2}{2}$$
$$= (V_2 - V_1)\left[a + \frac{b}{2}(V_1 + V_2)\right]$$
$$= 0.03\text{m}^3\left[-60 \text{ kN/m}^2 + \frac{7667}{2}\frac{\text{kN}}{\text{m}^5} \times 0.09\text{m}^3\right]$$
$$= 8.55 \text{ kJ} \quad \text{... (ii)}$$

work is done by the system, the magnitude being 8.55 kJ.

∴ Heat transfer involved is given by

From (i) and (ii)

$$Q_{1-2} = U_2 - U_1 + W_{1-2} = 59.5 + 8.55$$
$$Q_{1-2} = 68.05 \text{ kJ}$$

68.05 kJ of heat flow into the system during the process.

4. Open system analysis by first law

Level-1

4.1 Air flows steadily at the rate of 0.5 kg/s through an air compressor, entering at 7 m/s velocity, 100 kPa pressure, and 0.95 m^3/kg volume, and leaving at 5 m/s, 700 kPa, and 0.19 m^3/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in the compressor jacket absorbs heat from the air at the rate of 58 kW. (a) Calculate the rate of shaft work input to the air in kW (b) Find the ratio of the inlet pipe diameter to outlet pipe diameter.

(10 Marks)

Solution:

Given: $\dot{m} = 0.5$ kg; $C_1 = 7$ m/s, $P_1 = 100$ kPa, $v_1 = 0.95$ m^3/kg; $C_2 = 5$ m/s, $P_2 = 700$ kPa, $v_2 = 0.19$ m^3/kg

(a) Writing the steady flow energy equation, we have

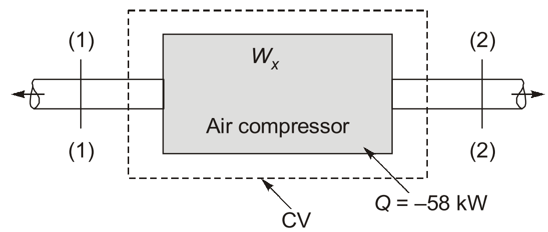
$$\dot{m}(u_1 + P_1v_1 + \frac{C_1^2}{2} + z_1g) + \frac{dQ}{dt} = \dot{m}(u_2 + P_2v_2 + \frac{C_2^2}{2} + z_2g) + \frac{dW_x}{dt}$$

$$\therefore \quad \frac{dW_x}{dt} = -\dot{m}\left[(u_2 - u_1) + (P_2v_2 - P_1v_1) + \frac{C_2^2 - C_1^2}{2} + (z_2 - z_1)g\right] + \frac{dQ}{dt}$$

$$\therefore \quad \frac{dW_x}{dt} = -0.5\left[90 + (7 \times 0.19 - 1 \times 0.95)100 + \frac{(5^2 - 7^2) + 10^{-3}}{2} + 0\right] - 58$$

$$= -0.5[90 + 38 - 0.012] - 58$$

$$\frac{dW_x}{dt} = -122 \text{ kW}$$



Rate of work input is 122 kW

(b) From mass balance, we have

$$\dot{m} = \frac{A_1C_1}{v_1} = \frac{A_2C_2}{v_2}$$

$$\frac{A_1}{A_2} = \frac{v_1}{v_2} \cdot \frac{C_2}{C_1} = \frac{0.95}{0.19} \times \frac{5}{7} = 3.57$$

$$\frac{d_1}{d_2} = \sqrt{3.57} = 1.89$$

4.2 In a steam power station, steam flows steadily through a 0.2 m diameter pipeline from the boiler to the turbine. At the boiler end, the steam conditions are found to be : P = 4 MPa, T = 400°C, h = 3213.6 kJ/kg, and v = 0.073 m^3/kg. At the turbine end the conditions are found to be : P = 3.5 MPa, T = 392°C, h = 3202.6 kJ/kg, and v = 0.084 m^3/kg. There is a heat loss of 8.5 kJ/kg from the pipeline. Calculate the steam flow rate.

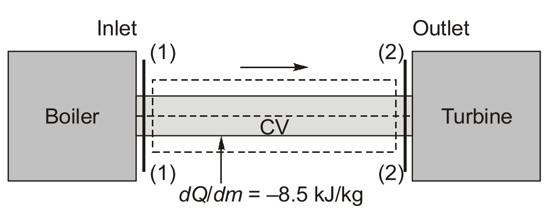
(10 Marks)

Solution:

Given: V_1 = Initial velocity, V_2 = Final velocity, v_1 = Specific volume at inlet, v_2 = Specific volume at outlet

The steady flow energy equation for the control volume as

$$h_1 + \frac{V_1^2}{2} + z_1g + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2} + z_2g + \frac{dW_x}{dm}$$



Here there is no change is datum, so change in potential energy will be zero.

Now,
$$\frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2}$$

$$V_2 = \frac{v_2}{v_1}.V_1 = \frac{0.084}{0.073}V_1 = 1.15V_1 = \frac{0.084}{0.073}V_1 = 1.15 \text{ V}$$

and $\frac{dW_x}{dm} = 0$

$$h_1 + \frac{V_1^2}{2} + \frac{dQ}{dm} = h_2 + \frac{V_2^2}{2}$$

$$\therefore \quad \frac{(V_2^2 - V_1^2)\times 10^{-3}}{2} = h_1 - h_2 + \frac{dQ}{dm}$$

$$= 3213.6 - 3202.6 - 8.5 = 2.5 \text{ kJ/kg}$$

$$V_1^2(1.15^2 - 1^2) = 5 \times 10^3$$

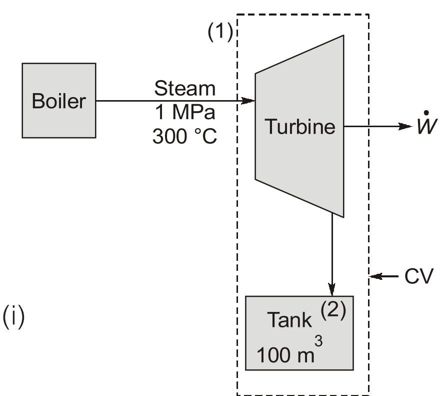
$$V_1^2 = 15{,}650 \text{ m}^2/\text{s}^2$$

$$V_1 = 125.1 \text{ m/s}$$

$$\therefore \quad \text{mass flow rate, } \dot{m} = \frac{A_1V_1}{v_1} = \frac{\frac{\pi}{4}\times(0.2)^2\times 125.1}{0.073} = 53.8 \text{ kg/s}$$

4.3 A boiler produces steam at 1 MPa and 300°C. The steam from the boiler is used to operate a turbine. The turbine exhausts steam into an evacuated tank of volume 100 m^3. The turbine operates till the pressure in the tank rises to 1 MPa at which point the temperature of steam in the tank is 250°C. Assuming that the turbine and tank are adiabatic, determine the work delivered by the turbine.
Use for steam: At 1 MPa and 250°C, v = 0.23195 m^3/kg, h = 2939.45 kJ/kg
At 1 MPa and 300°C, h = 3052.1 kJ/kg **(10 Marks)**

Solution:



$$v_2 = 0.23195 \text{ m}^3/\text{kg}$$
$$h_2 = 2939.45 \text{ kJ/kg}$$
$$h_i = 3052.1 \text{ kJ/kg}$$
$$u_2 = h_2 - Pv_2 = 2707.5 \text{ kJ/kg}$$

From 1st law of thermodynamics,

$$m_2u_2 - m_1u_1 = m_ih_i - m_eh_e + Q - W$$

since $m_1 = 0,\ m_e = 0,\ Q = 0$

$$\therefore \quad W = m_ih_i - m_2u_2 \quad \text{...(i)}$$

from mass conservation,

$$m_i = m_2$$

$$m_2 = \frac{V}{v_2} = \frac{100}{0.23195} = 431.127 \text{ kg}$$

From equation (i),

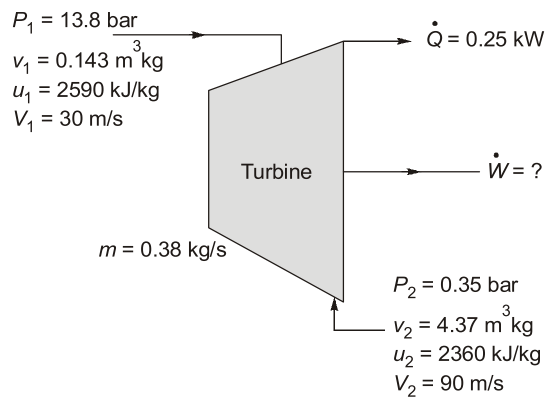
$$W = m(h_i - u_2) = 431.127(3052.1 - 2707.5)$$
$$W = 148{,}566.501 \text{ kJ} = 148.5665 \text{ MJ}$$

4.4 A turbine operating under steady flow conditions receives steam at the following state : pressure 13.8 bar; specific volume 0.143 m^3/kg; internal energy 2590 kJ/kg; velocity 30 m/s. The state of the steam leaving the turbine is : pressure 0.35 bar; specific volume 4.37 m^3/kg internal energy 2360 kJ/kg; velocity 90 m/s. Heat is lost to the surrounding at the rate of 0.25 kJ/s. If the rate of the steam flow is 0.38 kg/s, what is the power developed by the turbine?

(10 Marks)

Solution:

Given : P_1 = 13.8 bar, v_1 = 0.143 m^3/kg, u_1 = 2590 kJ/kg, V_1 = 30 m/s, m = 0.38 kg/s, $\dot{Q}$ = –0.25 kW,
P_2 = 0.35 bar, v_2 = 4.37 m^3/kg, u_2 = 2360 kJ/kg, V_2 = 90 m/s



From steady flow energy equation,

$$m_1\left(h_1+\frac{V_1^2}{2000}\right)+\dot{Q} = m_2\left(h_2+\frac{V_2^2}{2000}\right)+\dot{W}$$

$$0.38\left((2590+13.8\times100\times0.143)+\frac{30^2}{2000}\right)+(-0.25) = 0.38\left[(2360+0.35\times100\times4.37)+\frac{90^2}{2000}\right]+\dot{W}$$

$$1059.36+(0.25) = 956.46+\dot{W}$$

$$\dot{W} = 102.65 \text{ kW}$$

4.5 Air enters a frictionless adiabatic converging nozzle at 10 bar, 500 K with negligible velocity. The nozzle discharges to a region at 2 bar. If the exit area of the nozzle is 2.5 cm^2. Find the flow rate of air through the nozzle. Assume for air C_p = 1005 J/kgK and C_v = 718 J/kgK.

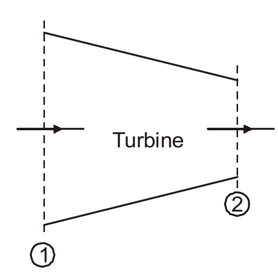
[1997 : 5 Marks]

Solution:

Since the process is adiabatic,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Applying SFEE,



$$h_1+\frac{C_1^2}{2} = h_2+\frac{C_2^2}{2}$$

$$\frac{C_2^2}{2} = h_1-h_2+\frac{C_1^2}{2}$$

$$\frac{C_2^2}{2} = c_P(T_1-T_2)+\frac{C_1^2}{2}$$

$$C_2 = \sqrt{2c_P(T_1-T_2)+C_1^2} = \sqrt{2\times1005(50-315.69)+0} \qquad [\because C_1=0]$$

$$C_2 = 608.66 \text{ m/s}$$

$\therefore$ Flow rate $= A_2\times C_2 = 2.5\times10^{-4}\times608.66$

$= 0.152 \text{ m}^3/\text{sec}$